METRIC SPACE (contd.)

07.2.26.

Open ball (or open sphere)

Let (X, d) be a metric space. Let $x_0 \in X$. Let r be any positive real number then the set of all those points of X which are within a distance of r from x_0 is called an open ball centred at x_0 and of radius r. It is denoted by $S(x_0, r)$ or $S_r(x_0)$ or $B(x_0, r)$ or $B_r(x_0)$.

Here, S stands for sphere and B stands for ball.

$$\therefore B(x_0, r) = \{x \in X : d(x, x_0) < r\}$$

It is also called r- neighbourhood (nbd) of x_0.

If $B[x_0, r] = \{x \in X : d(x, x_0) \leq r\}$

then B is called CLOSED BALL or CLOSED SPHERE with centre at x_0 and of radius r.

From definition, it is clear that every closed ball is an open ball but it is not necessary that every open ball is a closed ball.

Also, a ball is non-empty as it contains its centre x_0.